

Exercise 55

If f is a linear function, $f(0.1) = 11.5$, and $f(0.4) = -5.9$, find an equation for the function.

Solution

The general equation for a line is

$$y = mx + b.$$

The first equation says that when $x = 0.1$, $y = 11.5$.

$$11.5 = m(0.1) + b$$

The second point says that when $x = 0.4$, $y = -5.9$.

$$-5.9 = m(0.4) + b$$

This is a system of two equations with two unknowns that can be solved.

$$\begin{cases} 0.1m + b = 11.5 \\ 0.4m + b = -5.9 \end{cases}$$

Subtract the respective sides of these equations to eliminate b .

$$0.1m - 0.4m = 11.5 - (-5.9) \quad \rightarrow \quad -0.3m = 17.4 \quad \rightarrow \quad m = -58$$

Multiply both sides of the first equation by -4

$$\begin{cases} -0.4m - 4b = -46 \\ 0.4m + b = -5.9 \end{cases}$$

and then add the respective sides to eliminate m .

$$-4b + b = -46 + (-5.9) \quad \rightarrow \quad -3b = -51.9 \quad \rightarrow \quad b = 17.3$$

Now that m and b have been solved for, the line is known.

$$y = -58x + 17.3$$